

KÄNGURU DER MATHEMATIK 2023

16. 3. 2023



Level: Student, Grade: Schulstufe 11–13

Full name:

School:

Class:

Time: 75 min.

30 starting points

each correct answer to questions 1. – 10.: 3 points

each correct answer to questions 11. – 20.: 4 points

each correct answer to questions 21. – 30.: 5 points

each questions left unanswered: 0 points

each incorrect answer: minus $\frac{1}{4}$ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Zustimmungserklärung zur Datenverarbeitung für den österreichischen Wettbewerb „Känguru der Mathematik“

Mit meiner Unterschrift gebe ich das Einverständnis, dass meine angeführten personenbezogenen Daten (Vor- und Zuname, Klasse, Schulstufe, Schulstandort und Schulart) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen, sowie zur Erstellung und Veröffentlichung der Siegerlisten auf unserer Vereinshomepage (sofern mindestens 50 % der zu erreichenden Punktezahl erlangt werden bzw. ich unter den besten 10 einer Kategorie liege) verwendet werden dürfen.

Betroffenenrechte

Die Verwendung dieser Daten ist bis 31. Dezember des 2. Folgejahres gestattet. Nach diesem 31. Dezember werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei dieser durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art anonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage der DSGVO erlaubt.

Ich habe ein Recht auf Auskunft über meine gespeicherten personenbezogenen Daten, sowie das Recht auf Berichtigung, Datenübertragung, Widerspruch, Einschränkung der Bearbeitung sowie Sperrung oder Löschung unrichtig verarbeiteter Daten.

Ich kann die erteilte Einwilligung jederzeit auf der Homepage des Vereines Känguru der Mathematik unter www.kaenguru.at mittels des dafür bereitgestellten Formulars mit Wirkung für die Zukunft widerrufen (Art. 21 Abs. 1 DSGVO).

Ein Widerruf hat zur Folge, dass die personenbezogenen Daten nach gegenseitiger Rücksprache innerhalb von 31 Tagen gelöscht werden.

Durch den Widerruf wird die Rechtmäßigkeit der aufgrund der Einwilligung bis zum Widerruf erfolgten Verarbeitung nicht berührt. (Art. 7 Abs. 2 DSGVO)

Ort, Datum

Unterschrift



Information über den Känguruwettbewerb: www.kaenguru.at
Wenn du mehr in dieser Richtung machen möchtest,
gibt es die Österreichische Mathematikolympiade.
Infos unter: www.oemo.at

Känguru der Mathematik 2023
Level Student (Schulstufe 11, 12 and 13)
Austria – 16. 3. 2023



- 3 Point Examples -

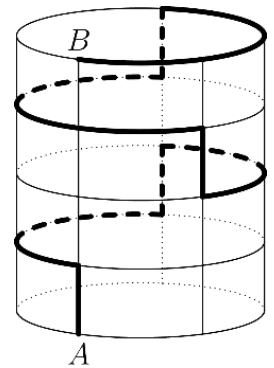
1. What is the simplified representation of the following fraction? $\frac{7777^2}{5555 \cdot 2222}$

- (A) 1 (B) $\frac{7}{10}$ (C) $\frac{49}{10}$ (D) $\frac{77}{110}$ (E) 49

2. Julia rolls 5 dice at the same time. She obtains a sum total of 19 points. What is the biggest number of sixes she can have rolled?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. A cylindrical tin is 15 cm high. The circumference of the base circle is 30 cm. An ant walks from point A at the base to point B at the top. Its path is partly vertically upwards and partly along horizontal circular arcs. Its path is drawn in bold on the diagram (with a solid line on the front and a dashed line at the back). How long is the total distance covered by the ant?

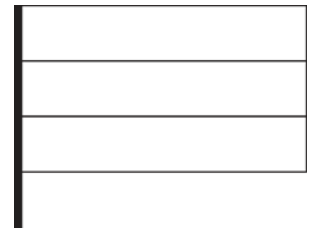


- (A) 45 cm (B) 55 cm (C) 60 cm (D) 65 cm (E) 75 cm

4. Let A be a 2023-digit number where every digit is 1. What is the sum of the digits of the number $A \cdot 1111$?

- (A) 8080 (B) 8083 (C) 8086 (D) 8092 (E) 8101

5. Emma should colour in the three strips of the flag shown. She has four colours available. She can only use one colour for each strip and immediately adjacent strips are not to be of the same colour.



How many different ways are there for her to colour in the flag?

- (A) 24 (B) 27 (C) 32 (D) 36 (E) 64

6. We call a positive integer n *twoprime*, if it has exactly three different positive factors, namely 1, 2 and the number n itself. How many twoprime numbers are there?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. What is the units digit of the following product? $(5^5 + 1) \cdot (5^{10} + 1) \cdot (5^{15} + 1)$

- (A) 0 (B) 1 (C) 3 (D) 5 (E) 6

8. What is the value of the following sum?

$$2^{0^{2^3}} + 0^{2^{3^2}} + 2^{3^{2^0}} + 3^{2^{0^2}}$$

- (A) 3 (B) 4 (C) 7 (D) 12 (E) more than 100

9. 23 animals are sitting in the first row of a cinema. Each animal is either a beaver or a kangaroo. Each animal has at least one kangaroo next to it.

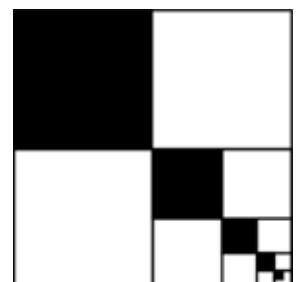
What is the maximum amount of beavers in the row?

- (A) 7 (B) 8 (C) 10 (D) 11 (E) 12

10. A square with area 84 is split into four squares. The upper left square is coloured in black. The lower right square is again split into four squares and so on. The process is repeated infinitely many times.

How big is the area coloured in black?

- (A) 24 (B) 28 (C) 31 (D) 35 (E) 42

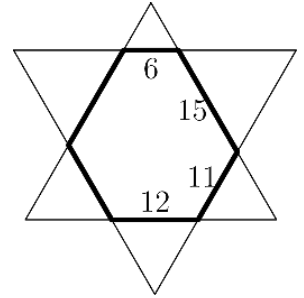


- 4 Point Examples -

7	9					
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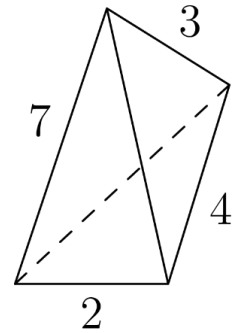
11. The numbers from 1 to 9 are to be distributed to the nine squares in the diagram according to the following rules: There is to be one number in each square. The sum of three adjacent numbers is always a multiple of 3. The numbers 7 and 9 are already written in. How many ways are there to insert the remaining numbers?

- (A) 9 (B) 12 (C) 15 (D) 18 (E) 24



12. Two equilateral triangles of different sizes are placed on top of each other so that a hexagon is formed on the inside whose opposite sides are parallel. Four of the side lengths of the hexagon are stated in the diagram. How big is the perimeter of the hexagon?

- (A) 64 (B) 66 (C) 68 (D) 70 (E) 72



13. Consider the five numbers a_1, a_2, a_3, a_4, a_5 with sum S . It is known that $a_k = k + S$ for $1 \leq k \leq 5$. What is the value of S ?

- (A) $\frac{15}{4}$ (B) $-\frac{15}{4}$ (C) -15 (D) 15 (E) another number

14. In a three-sided pyramid all side lengths are integers. Four of the side lengths can be seen in the diagram. What is the sum of the two remaining side lengths?

- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

15. How many pairs of integers (m, n) fulfil the inequality $|2m - 2023| + |2n - m| \leq 1$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. The number 5^{5^6} is to be written in the form n^n where n is a natural number. What is the value of n ?

- (A) $5 \cdot 5^4$ (B) $5 \cdot 5^5$ (C) 5^{30} (D) 30 (E) 25^{25}

17. Leon has drawn a closed path on the surface of a cuboid.

Which net can represent his path?

(A)

(B)

(C)

(D)

(E)

18. For each positive integer n the number $n!$ is defined as the product of all numbers from 1 to n . For example, $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$. For a certain N the formula $N! = 6! \cdot 7!$ holds.

How big is the sum of the digits of N ?

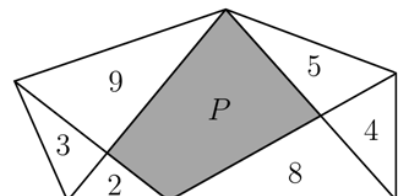
- (A) 1 (B) 2 (C) 4 (D) 8 (E) 9

19. The graphs of the functions $y = x^3 + 3x^2 + ax + 2a + 4$ all pass through a common point independent of the choice of a . How big is the sum of the co-ordinates of this common point?

- (A) 2 (B) 4 (C) 7 (D) 8 (E) another number

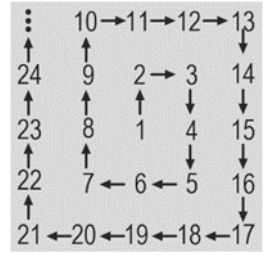
20. A pentagon is cut into smaller parts as shown in the diagram. The numbers in the triangles state the area of the according triangle. How big is the area P of the grey quadrilateral?

- (A) 15 (B) $\frac{31}{2}$ (C) 16 (D) 17 (E) another number



- 5 Point Examples -

21. The diagram shows a spiral of consecutive numbers starting with 1. In which order will the numbers 625, 626 and 627 appear in the spiral?

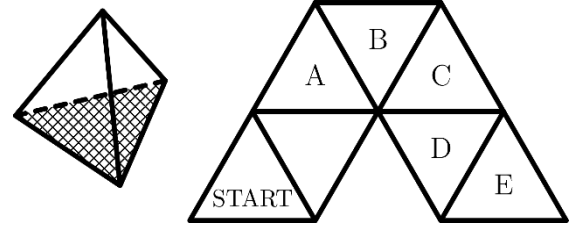


- (A) (B) (C) (D) (E)

22. How many positive integers divide $2^{20} \cdot 3^{23}$ but not $2^{10} \cdot 3^{20}$?
 (A) 13 (B) 30 (C) 273 (D) 460 (E) another number

23. 13 athletes took part in a three-part climbing competition. There are no draws in any part. The final rank of each athlete is determined by arranging the products of the ranks in each of the three parts: If an athlete for example comes 4th once, 3rd once and 6th once, he has $4 \cdot 3 \cdot 6 = 72$ points. The higher the number of points, the worse the final rank. What is the worst possible final rank Hans can get to if he was 1st in two of the parts?
 (A) 2. (B) 3. (C) 4. (D) 5. (E) 6.

24. A game marker in the shape of a regular tetrahedron has one marked area. That side is placed on the triangle marked START. The marker is then moved within the diagram always to the next adjacent triangle by rolling it around an edge. On which triangle is the marker when it is on the marked side again for the first time?



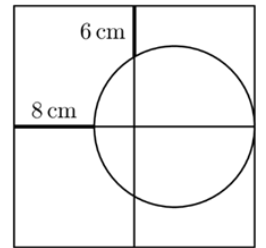
- (A) A (B) B (C) C (D) D (E) E

25. A part of a polynomial of degree five is illegible due to an ink stain. It is known that all zeros of the polynomial are integers. What is the highest power of $x-1$ that divides this polynomial?



- (A) $(x-1)^1$ (B) $(x-1)^2$ (C) $(x-1)^3$ (D) $(x-1)^4$ (E) $(x-1)^5$

26. The big square shown is split into four small squares. The circle touches the right side of the square in its midpoint. How big is the side length of the big square? (Hint: The diagram is not drawn to scale.)

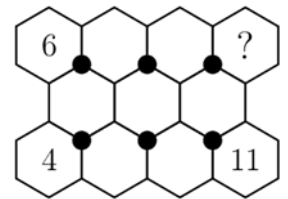


- (A) 18 cm (B) 20 cm (C) 24 cm (D) 28 cm (E) 30 cm

27. What is the biggest common factor of all numbers of the form $n^3 \cdot (n+1)^3 \cdot (n+2)^3 \cdot (n+3)^3 \cdot (n+4)^3$ where n is a positive integer?

- (A) $2^9 \cdot 3^3$ (B) $2^3 \cdot 3^3 \cdot 5^3$ (C) $2^6 \cdot 3^3 \cdot 5^3$ (D) $2^8 \cdot 3^2 \cdot 5^3$ (E) $2^9 \cdot 3^3 \cdot 5^3$

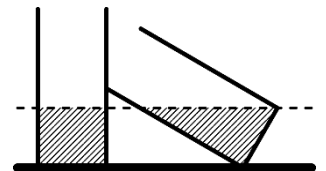
28. The numbers from 1 to 11 are written in the empty hexagons. The sums of the three numbers in three hexagons with a common bold point are always equal. Three of the eleven numbers are already written in (see diagram).



Which number is written in the hexagon with the question mark?

- (A) 5 (B) 4 (C) 7 (D) 3 (E) 9

29. Two identical cylindrical glasses contain the same amount of water. The left glass is upright, while the right one rests against the other one at a slant. The water level in both glasses is at the same height. The water level in the leaning glass touches its bottom in exactly one point (see diagram). The bases of both glasses have an area of 3π cm². How much water is in each glass?



- (A) 9π cm³ (B) 6π cm³ (C) $3\sqrt{3}\pi$ cm³ (D) $\frac{3\pi}{4}$ cm³ (E) It cannot be uniquely determined from this information.

30. The product of six consecutive numbers is a 12-digit number of the form $abb\ cdd\ cdd\ abb$, where the digits a , b , c and d are also consecutive numbers in any order.

What is the value of the digit d ?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5