

Mathematical Kangaroo 2015

Group Student (Grade 11 and above)

Austria - 23. 3. 2015



- 3 point questions -

1. Andrea was born sometime in the year 1997 and her sister Charlotte sometime in the year 2001. What is known for certain about the age difference of the two sisters? It is...

- (A) less than 4 years (B) at least 4 years (C) exactly 4 years
 (D) more than 4 years (E) not less than 3 years

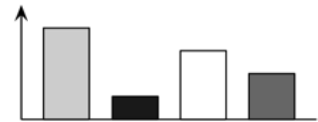
2. $(a - b)^3 + (b - a)^3 =$

- (A) 0 (B) $2(a - b)^3$ (C) $2a^3 - 2b^3$ (D) $2a^3 + 2b^3$ (E) $2a^3 + 6a^2b + 6ab^2 + 2b^3$

3. How many real solutions has the equation $2^{2x} = 4^{x+1}$?

- (A) 0 (B) infinitely many (C) 2 (D) 1 (E) 3

4. Diana produces a bar chart which shows the number of four different types of trees which she has counted on a biology trip. Heinz believes that a pie chart would represent the ratio of the different types of trees in a better way. What would the pie chart look like?

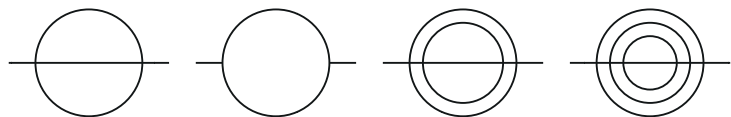


- (A) (B) (C) (D) (E)

5. If you add all the whole numbers from 2001 to 2031 and then divide the sum by 31, you get;

- (A) 2012 (B) 2013 (C) 2015 (D) 2016 (E) 2496

6. How many of the following shapes can be drawn using one continuous line (i.e. without lifting the pencil) and without going over a line twice?

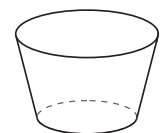


- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. A quadrilateral is called convex if all its internal angles are less than 180° . The number of right angles in a convex quadrilateral is n . Which of the following lists is a complete listing of all possible values of n ?

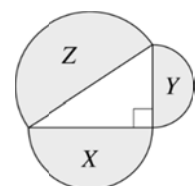
- (A) 0, 1, 2 (B) 0, 1, 2, 4 (C) 0, 1, 2, 3, 4 (D) 0, 1, 3 (E) 1, 2, 3

8. A drinking glass is made in the shape of a truncated cone. The outside of the glass (without the upper or lower circle) should be covered with coloured paper. How do you need to cut the paper to completely cover the glass without an overlap?



- (A) (B) (C) (D) (E)

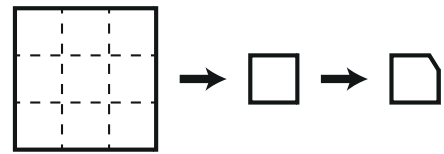
9. The diameters of three semi-circles form the sides of a right-angled triangle. Their areas are $X \text{ cm}^2$, $Y \text{ cm}^2$ and $Z \text{ cm}^2$ as pictured. Which of the following expressions is definitely correct?



- (A) $X + Y < Z$ (B) $\sqrt{X} + \sqrt{Y} = \sqrt{Z}$ (C) $X + Y = Z$ (D) $X^2 + Y^2 = Z^2$ (E) $X^2 + Y^2 = Z$

10. A square bit of paper is folded along the dashed lines in some order and direction. One of the corners of the resulting small square is cut off. The piece of paper is then unfolded. How many holes are on the inner area of the piece of paper?

- (A) 0 (B) 1 (C) 2 (D) 4 (E) 9



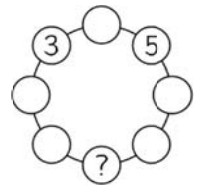
- 4 point questions -

11. $\sqrt{(2015 + 2015) + (2015 - 2015) + (2015 \times 2015) + (2015 \div 2015)} =$
 (A) $\sqrt{2015}$ (B) 2015 (C) 2016 (D) 2017 (E) 4030

12. The x -axis and the graphs of $f(x) = 2 - x^2$ and $g(x) = x^2 - 1$ split the co-ordinate plane into
 (A) 7 regions (B) 8 regions (C) 9 regions (D) 10 regions (E) 11 regions

13 Ella wants to write a number into each circle in the diagram on the right, in such a way that each number is equal to the sum, of its two direct neighbours. Which number does Ella need to write into the circle marked with „?“.

- (A) -5 (B) -16 (C) -8 (D) -3 (E) This question has no solution.



14. We know the following about five positive whole numbers a, b, c, d, e . All the numbers are different, $b = c : e$, $d = a + b$ and $a = e - d$. Which of the numbers a, b, c, d, e is the largest?

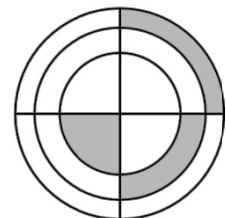
- (A) a (B) b (C) c (D) d (E) e

15. The geometric mean of n numbers is defined as the n^{th} root of the product of all n numbers, that is $\sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$. We have six numbers. The geometric mean of three of them is 3, the geometric mean of the other three is 12. How big is the geometric mean of all six numbers?

- (A) 4 (B) 6 (C) $\frac{15}{2}$ (D) $\frac{15}{6}$ (E) 36

16. The diagram shows three concentric circles and two perpendicular, common diameters of the three circles. The three grey sections are of equal area, the small circle has radius 1. What is the product of the radii of the three circles?

- (A) $\sqrt{6}$ (B) 3 (C) $\frac{3\sqrt{3}}{2}$ (D) $2\sqrt{2}$ (E) 6



17. In the past 20 years the population of Arnberg has increased by 40%. In the same time span the population of Berghausen has increased by 60%. In total the population of the two villages has increased by 54%. What was the ratio of the populations 20 years ago?

- (A) 10:13 (B) 20:27 (C) 3:7 (D) 7:12 (E) 2:3

18. Bibi rolls a die which has the numbers 1, 2, 3, 4, 5, 6 on its faces. At the same time Tina rolls a die which has the numbers 2, 2, 2, 5, 5, 5 on its faces. Tina wins if she rolls a number higher than Bibi. What is the probability that Tina wins?

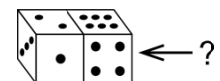
- (A) $\frac{1}{3}$ (B) $\frac{7}{18}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$ (E) $\frac{11}{18}$

19 There are 2015 marbles in a pipe. They are numbered 1 to 2015. Marbles whose digits add up to the same number, have the same colour and marbles whose digits have a different sum, have a different colour. How many different colours do the marbles in the pipe have?

- (A) 10 (B) 27 (C) 28 (D) 29 (E) 2015

20. On a standard die the sum of the numbers on opposite faces is always 7. Two identical standard dice are shown in the figure. How many dots could there be on the non-visible right-hand face (marked with “?”)?

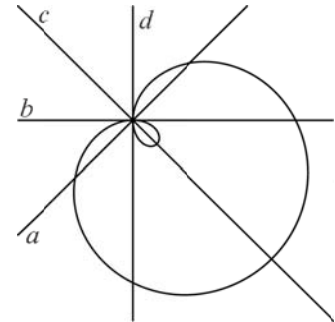
- A) only 5 (B) only 2 (C) either 2 or 5 (D) either 1, 2, 3 or 5 (E) either 2, 3 or 5



- 5 point questions -

21. Die Aussagen (A) – (E) werden der Reihe nach auf ihre Wahrheit überprüft. Welche davon ist die erste wahre Aussage?

- (A) (C) is true. (B) (A) is true. (C) (E) is false.
 (D) (B) is false. (E) $1 + 1 = 2$



22. The curve in the diagram is defined by the equation

$$(x^2 + y^2 - 2x)^2 = 2(x^2 + y^2)$$

Which of the lines a, b, c, d is the y -axis?

- (A) a (B) b (C) c (D) d (E) none of them

23. The following table is the multiplication table of the numbers 1 to 10.

What is the sum of all 100 products in the complete table?

- (A) 1000 (B) 2025 (C) 2500 (D) 3025 (E) 5500

•	1	2	3	...	10
1	1	2	3	...	10
2	2	4	6	...	20
3	3	6	9	...	30
...
10	10	20	30	...	100

24. How many regular n -sided shapes are there, whose angles (in degrees) are whole numbers?

- (A) 17 (B) 18 (C) 22 (D) 25 (E) 60

25. How many three-digit whole numbers can be written as the sum of exactly nine different powers of two? (Hint: Powers of two are $2^0, 2^1, 2^2, 2^3, \dots$)

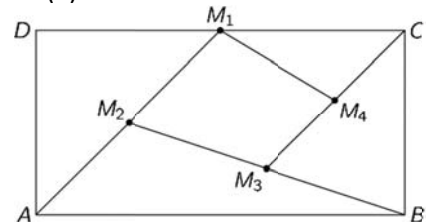
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

26. How many different triangles ABC whose side lengths are whole numbers are there, if $\angle ABC = 90^\circ$ and $AB = 20$? (Hint: Two triangles are called different if they are not congruent.)

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 6

27. In the rectangle $ABCD$ pictured, M_1 is the midpoint of DC , M_2 the midpoint of AM_1 , M_3 the midpoint of BM_2 and M_4 the midpoint of CM_3 . Determine the ratio of the area of the quadrilateral $M_1M_2M_3M_4$ to the area of the rectangle $ABCD$.

- (A) $\frac{7}{16}$ (B) $\frac{3}{16}$ (C) $\frac{7}{32}$ (D) $\frac{9}{32}$ (E) $\frac{1}{5}$



28. On a board there are blue and red rectangles. Exactly 7 of the rectangles are squares. There are 3 more red rectangles than blue squares. There are also two more red squares than blue rectangles. How many blue rectangles are there on the board?

- (A) 1 (B) 3 (C) 5 (D) 6 (E) 10

29. The 96 members of a counting club are standing in a circle counting. They start with 1, 2, 3, etc., each person in the circle saying the next number in turn. If a member of the club says an even number, he steps out of the circle. The remaining members continue, starting the second round with 97. They continue in this way until only one member of the club is left. Which number did this person say in round one?

- (A) 1 (B) 17 (C) 33 (D) 65 (E) 95

30. Independently from each other Bill and Bob substitute the letters in the word KANGAROO with numbers, so that the resulting numbers are multiples of 11. They both substitute different letters with different digits and same letters with the same digits ($K \neq 0$). Bill obtains the biggest possible number and Bob the smallest possible. In both cases one letter is substituted with the same digit. Which digit is that?

- (A) 0 (B) 3 (C) 4 (D) 5 (E) 6