MATHEMATICS KANGAROO 2011
Austria - 17.3.2011

Group: Junior, Grades: 9-10

Name:	
School:	
Class:	

Time allowed: 75 min.
Each correct answer, questions 1.-10.: 3 Points
Each correct answer, questions 11.-20.: 4 Points
Each correct answer, questions 21.-30.: 5 Points
Each question with no answer given: 0 Points
Each incorrect answer: Lose ¼ oft he points for that question.
You begin with 30 points.

Please write the letter (A, B, C, D, E) of the correct answer under the question number (1 to 30). Write neatly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Information über den Känguruwettbewerb: www.kaenguru.at
Wenn Du mehr in dieser Richtung machen möchtest, gibt es die Österreichische Mathematikolympiade; Infos unter: www.oemo.at

Ich melde mich zur Teilnahme zum österreichischen Wettbewerb „Känguru der Mathematik 2011“ an.
Ich stimme zu, dass meine personenbezogenen Daten, nämlich Vor- und Zuname, Geschlecht, Klasse, Schulstufe, Schulstandort und Schulart zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von landes- sowie österreichweiten Reihungen, der Veröffentlichung der Ergebnisse jener Schülerinnen und Schüler, die in ihrer Kategorie zumindest 50% der zu vergebenden Punkte erreicht haben sowie des Ermöglichens von Vergleichen mit eigenen Leistungen aus vorherigen Wettbewerbsperioden auf www.kaenguru.at. bzw. http://kaenguru.diefenbach.at/ verwendet werden.
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Nach dem 31. Dezember 2013 werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei das zuletzt genannte Datum durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art pseudonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage von § 46 Abs. 1 Z 3 DSG 2000 erlaubt.

Unterschrift:

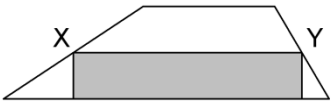
Mathematics Kangaroo 2011
Group Junior (Grade 9 and 10)
Austria - 17.3.2011

- 3 Point Questions -

1) A zebra crossing has alternating white and black stripes each 50cm wide. The first stripe is white and the last one is white. The zebra crossing in front of our school has 8 white stripes. How wide is the road?

A) 7 m **B)** 7.5 m **C)** 8 m **D)** 8.5 m **E)** 9 m

2) The area of the grey rectangle shown on the right is 13 cm^2. X and Y are the midpoints of the sides of the trapezium. How big is the area of the trapezium?

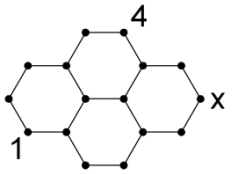


A) 24 cm^2 **B)** 25 cm^2 **C)** 26 cm^2 **D)** 27 cm^2 **E)** 28 cm^2

3) Given are the following expressions $S_1 = 2\times3+3\times4+4\times5$, $S_2 = 2^2+3^2+4^2$, $S_3 = 1\times2+2\times3+3\times4$. Which one of the following statements is true?

A) $S_2 < S_1 < S_3$ **B)** $S_1 < S_2 = S3$ **C)** $S_1 < S_2 < S_3$ **D)** $S_3 < S_2 < S_1$ **E)** $S_1 = S_2 < S_3$

4) In the picture on the right a number should be written next to each point. The sum of the numbers on the corners of each side of the hexagon should be equal. Two numbers have already been written. Which number should be in the place marked 'x'?



A) 1 **B)** 3 **C)** 4 **D)** 5 **E)** 24

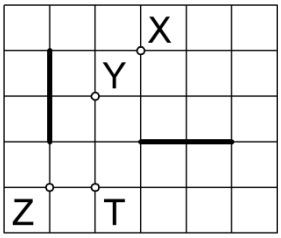
5) If 2011 is divided by a certain positive whole number the remainder is 1011. Which number was it divided by?

A) 100 **B)** 500 **C)** 1000 **D)** another number **E)** There is no such number.

6) A rectangle with area 360 cm^2 is being laid out with square tiles. The rectangle is 24 cm long and 5 tiles wide. How big is the area of one tile in cm^2?

A) 1 **B)** 4 **C)** 9 **D)** 16 **E)** 25

7) All four-digit numbers whose digit sum is 4 are written down in order of size, starting with the biggest. In which position is the number 2011??

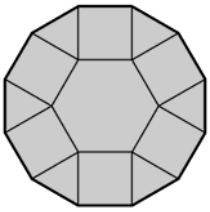


A) 6. **B)** 7. **C)** 8. **D)** 9. **E)** 10.

8) The two bold lines on the right are rotations of each other. Which of the given points could be the centre of this rotation?

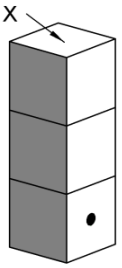
A) only *X* **B)** *X* and *Z* **C)** *X* and *T* **D)** only *T* **E)** *X, Y, Z* and *T*

9) Given are a regular hexagon with side-length 1, six squares and six equilateral triangles as shown on the right. What is the perimeter of this tessellation?



A) $6(1+\sqrt{2})$ **B)** $6\left(1+\frac{\sqrt{3}}{2}\right)$ **C)** 12 **D)** $6+3\sqrt{2}$ **E)** 9

10) In the picture on the left we see three dice on top of each other. The sum of the points on opposite sides of the dice is 7 as usual. The sum of the points of areas that face each other is always 5. How many points are on the area marked X?



A) 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

- 4 Point Questions -

11) In a certain month there were 5 Mondays, 5 Tuesdays and 5 Wednesdays. In the month before there were only 4 Sundays. What will be in next month?

A) exactly 4 Fridays **B)** exactly 4 Saturdays **C)** 5 Sundays
D) 5 Wednesdays **E)** This situation is impossible.

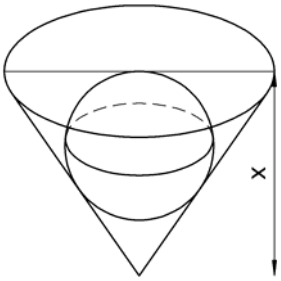
12) Three racers take part in a Formula-1 Race: Michael, Fernando and Sebastian. From the start Michael is in the lead in front of Fernando who is in front of Sebastian. In the course of the race Michael and Fernando overtake each other 9 times, Fernando and Sebastian 10 times and Michael and Sebastian 11 times. In which order do those three end the race?

A) Michael, Fernando, Sebastian **B)** Fernando, Sebastian, Michael
C) Sebastian, Michael, Fernando **D)** Sebastian, Fernando, Michael
E) Fernando, Michael, Sebastian

13) How big is n, if $9^n + 9^n + 9^n = 3^{2011}$ holds true?

A) 1005 **B)** 1006 **C)** 2010 **D)** 2011 **E)** another value

14) I have got two cubes with side lengths *a* dm and *a +1* dm. The big cube is full of water and the little one is empty. I pour as much water as possible from the big one into the little one and now 217 ℓ remain in the big die. How many litres of water are now in the little one?

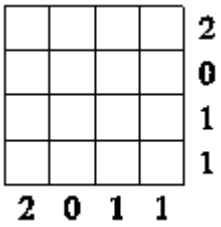


A) 243 ℓ **B)** 512 ℓ **C)** 125 ℓ **D)** 1331 ℓ **E)** 729 ℓ

15) A marble of radius 15 is rolled into a cone-shaped hole. It fits in perfectly. From the side the cone looks like an equilateral triangle. How deep is the hole?

A) $30\sqrt{2}$ **B)** $25\sqrt{3}$ **C)** 45 **D)** 60 **E)** $60\ (\sqrt{3}-1)$

16) The cells of the 4×4-table on the right should be coloured either in black or white. The numbers determine how many cells in each row/column should be black. How many ways are there to do the colouring in?

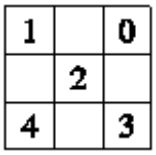


A) 0 **B)** 1 **C)** 3 **D)** 5 **E)** 9

17) What is the biggest number of consecutive three-digit numbers with at least one odd digit each?

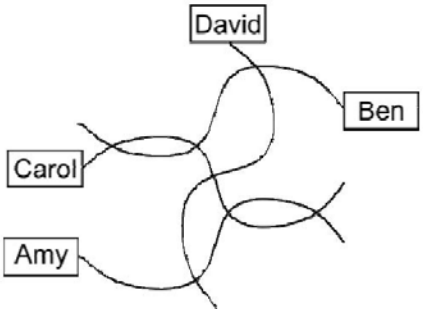
A) 1 **B)** 10 **C)** 110 **D)** 111 **E)** 221

18) Nick wants to write whole numbers into the cells of the 3×3-table on the right so that the sum of the digits in each in each 2×2-sub-table is always 10. Five numbers have already been written. Determine the sum of the remaining four numbers.



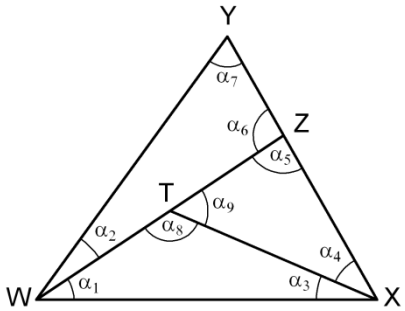
A) 0 **B)** 10 **C)** 11 **D)** 12 **E)** 13

19) Jan cannot draw very accurately but nevertheless he tried to produce a road map of his village. The relative position of the houses and the street crossings are all correct but three of the roads are actually straight and only the Qurwikroad is not. Who lives in the Qurwikroad?



A) Amy **B)** Ben **C)** Carol **D)** David
E) It cannot be determined from the drawing.

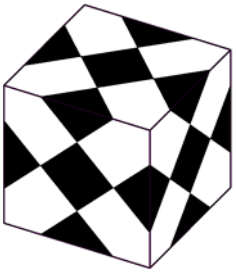
20) In the triangle WXY points Z on XY and T on WZ are, as shown on the right. If one connects T with X, a figure with nine internal angles is created as shown in the figure on the right. From those 9 angles, what is the smallest number that could be a different size to each other



A) 2 **B)** 3 **C)** 4 **D)** 5 **E)** 6

- 5 Point Questions-

21) Simon has a cube with side length 1 dm made of glass. He sticks several equally big black squares on it, as shown on the right so that all faces look the same. How many cm^2 were covered over?



A) 37.5 **B)** 150 **C)** 225 **D)** 300 **E)** 375

22) The five-digit number *abcde* is called *interesting*, if all of its digits are different and $a = b+c+d+e$ holds true. How many interesting numbers are there?

A) 72 **B)** 144 **C)** 168 **D)** 216 **E)** 288

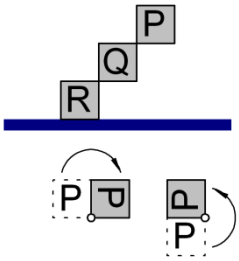
23) The numbers x and y are both greater than 1. Which of the following numbers is biggest?

A) $\frac{x}{y+1}$ **B)** $\frac{x}{y-1}$ **C)** $\frac{2x}{2y+1}$ **D)** $\frac{2x}{2y-1}$ **E)** $\frac{3x}{3y+1}$

24) Given is a regular tetrahedron ABCD whose side ABC is on the plane ε. The edge BC is on the straight line s. Another tetrahedron BCDE has one common side with ABCD. Where does the straight line DE intersect the plane ε?

A) Inside of ABC, on the same side of s as A.
B) Outside of ABC, on the same side of s as A.
C) Outside of ABC, not on the same side of s as A.
D) DE is parallel to ε.
E) The answer depends on the side length of the tetrahedrons.

25) Three big boxes P, Q and R are stored in a warehouse. The upper picture on the right shows their placements from above. The boxes are so heavy that they can only be rotated 90° around a vertical edge as indicated in the pictures below. Now the boxes should be rotated to stand against the wall in a certain order. Which arrangement is possible?



A) 

B) 

C) 

D) 

E) All four arrangements are possible.

26) How many ordered pairs of positive whole numbers (x, y) solve the equation $\frac{1}{x}+\frac{1}{y}=\frac{1}{3}$?

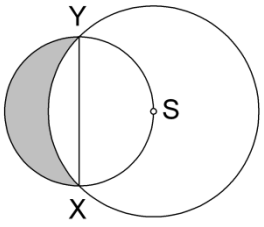
A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** 4

27) For a positive whole number $n \geq 2$ let $\langle n \rangle$ indicate the largest prime number less than or equal to *n*.

How many positive whole numbers *k* fulfill the condition $\langle k+1 \rangle + \langle k+2 \rangle = \langle 2k+3 \rangle$?

A) 0 **B)** 1 **C)** 2 **D)** 3 **E)** more than 3

28) The two circles shown on the right intersect each other at X and Y. Thereby XY is the diameter of the small circle. The centre S of the large circle (with radius r) is on the small circle. How big is the area of the grey region?



A) $\frac{\pi}{6}r^2$ **B)** $\frac{\sqrt{3}\pi}{12}r^2$ **C)** $\frac{1}{2}r^2$ **D)** $\frac{\sqrt{3}}{4}r^2$ **E)** another number

29) In how many ways can one choose four edges of a cube so that no two of these edges have a common corner?

A) 6 **B)** 8 **C)** 9 **D)** 12 **E)** 18

30) Determine all *n* ($1 \leq n \leq 8$) for which one can mark several cells of a 5×5 table so that there are exactly *n* marked cells in every 3×3 subtable.

A) 1 **B)** 1 and 2 **C)** 1, 2 and 3 **D)** 1, 2, 7 and 8
E) All numbers from 1 to 8 are possible.